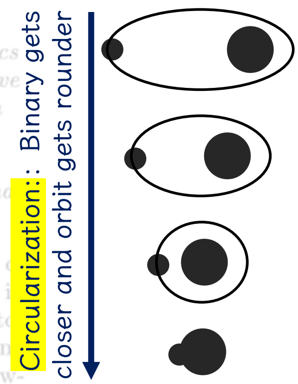
Once we have space-based gravitational wave detectors, we will know eccentricities of those binary black hole systems, and then know how they are formed

中文版
点这里

Massive thanks to my supervisor and collaborators (ˋ • ω • ˊ)



More circular, less eccentric. e=0 for circular orbit

TL; DR:

- How those stellar-mass binary black holes (sBBHs) are formed? Orbital eccentricity is the key
- But it is not enough if rely on ground detectors only
- Turn to space detectors? Still have their own challenge on this
- Embrace both sides! We did a kind of multiband search called "archival search", then the detection could cost less
- Having said this, it still incurs additional computational cost when bring in the eccentricity, which should let folks be aware of
- Anyway, we first implemented a real archival search process and also estimated how much burden was added

I. From the very first beginning to what we gonna do

Many sBBHs have been detected so far since 2015, well done LIGO/Virgo! But astronomers are even more confused about how these systems formed.

The eccentricity can be the key, but the systems we've seen so far are all (almost) circular.

Why? The closer black holes get, the more energy they lose, the more circular their orbits become. Almost no eccentricity retains by the merger time. That's the limitation of ground detectors.

So how can we see the pre-merger signals?

Space detectors: Here I am to help!

sBBHs are within the sensitive range of space detectors during the inspiral phase, before they are fully circularized.

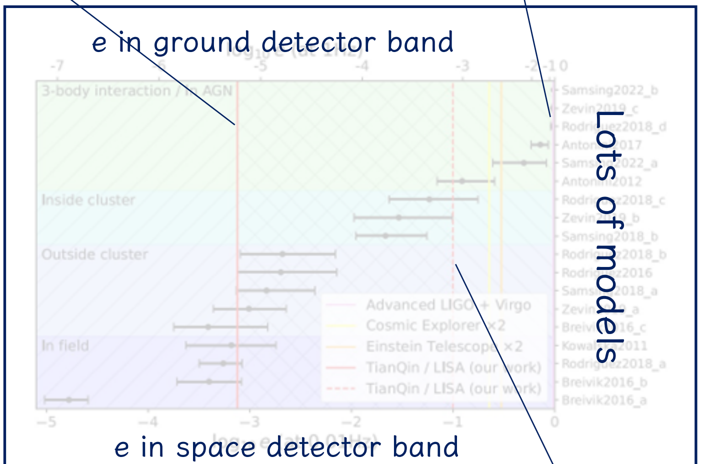
See Fig 1↓! Astronomers have done various models, such as:

- sBBHs are isolated, no one bothers them, then the orbits will tend to be circular;
- If they're in a busy place like a globular cluster, the eccentricity is quite a bit bigger;
- But if they get thrown out of the cluster halfway through merger, it's not much different from isolated evolution;
- It 's even more crowded if you consider active galactic nuclei or something, then the eccentricity will be very close to 1.

*Electronic address: huyiming@mail.sysu.edu.cn

Our work proves:
Space detectors can distinguish smaller e

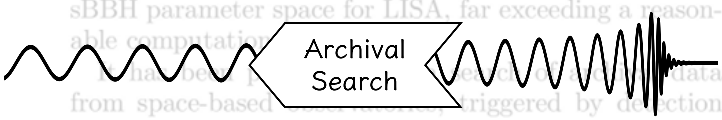
The line of LIGO is almost on the axis, so many models to the left waiting to be distinguished



Our work: Because of resource constraints we go this big for now, but that doesn't mean to be the upper limit of the space detectors

Fantastic! So how exactly should we detect them? Match filtering.

It's like recognizing a song by listening to it, first you need to have a template bank that cover a wide range of songs. Ground detector signals lasts for a few seconds, 100,000 is enough, but space detector signals can be several years, then we need 10^{30}, which is really terrible!



Then someone came up with an idea: why not look at the merger signals given from the ground, and then go to the archival data of space detectors to see if you can find them, i.e., archival search.

This is very targeted, most of the information is already in hand, we just need a small template bank for the space detectors.

Nice idea, but no one really implemented this yet, let alone take the eccentricity into account.

Now it is our turn ↓

II. Technical details

To build up a template bank we need a waveform, we choose one called EccentricFD

For space detectors we consider TianQin (天琴) and LISA. They are both planned to work in 2030s.

So the next generation ground detectors like *Cosmic Explorer (CE)* and *Einstein Telescope (ET)* are really the detectors for archival searches

But CE and ET haven't really started working yet either, so we need to find a way to estimate their detection capability: A tool called the Fisher Information Matrix.

After calculating we are glad to find it was similar to the conclusions of previous work.

Ground detectors can measure most of the parameter more accurately more than space detectors, but not these two:

One is the previously mentioned eccentricity; the other is the dominant parameter chirp mass M.

So we will fix all the other parameters and generate template banks on these two parameters.

What's going on inside? Check this GitHub repo: [HumphreyWang/sbank_simplified](https://github.com/HumphreyWang/sbank_simplified)

So what do you use to generate a template bank? **sbank**, a Python package.

In reality, it's impossible for the data to be identical to the template, so how do we assess whether they are matched or not?

$$\mathrm{FF}\left(\lambda^{\mu}\right) \equiv \max_{\lambda^{\mu'}} \frac{\left(h(\lambda^{\mu})\middle|h(\lambda^{\mu'})\right)}{\sqrt{\left(h(\lambda^{\mu})|h(\lambda^{\mu})\right)\left(h(\lambda^{\mu'})|h(\lambda^{\mu'})\right)}}. \quad (1)$$

Here $\lambda^{\mu'}$ denotes the parameter set for a template in the bank, and λ^{μ} is the parameter set for the test waveform.

Good question, that's what the fitting factor FF is for. We usually take 0.97 as the threshold, anything lower than that means they doesn't match, and your template bank fails to find that signal.

But don't hurry, there are still several things to keep in mind, space detectors have to take into account the antenna response, harmonics should also be considered because of the eccentricity, and ...... It's a lot more trouble than one might expect!

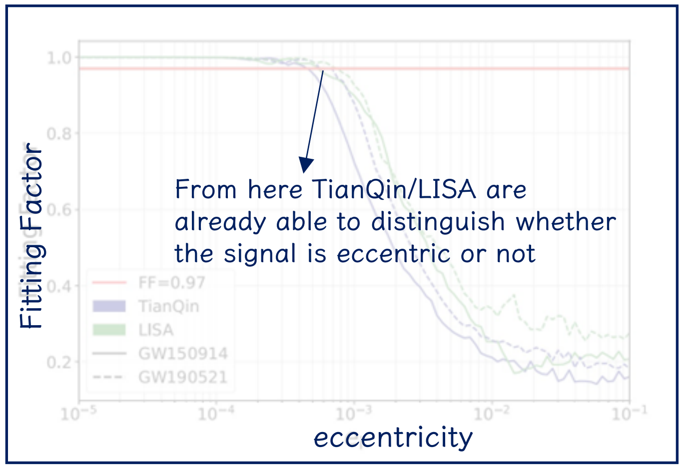


FIG. 2: The fitting factor between a noneccentric template bank and a signal with different eccentricities. The blue(green) lines denote the banks of TianQin(LISA), the solid(dashed) lines correspond to the banks of a GW150914-like(GW190521-like) scenario.

III. Template bank! Here we go!

Wait a minute! With all the difficulties mentioned above, why don't we just FORGET about the eccentricity?

Then we have to see how small an e can we ignore, see Fig 2↑: Use a non-eccentric template bank to match signals with a variety of eccentricity. Just a small e it won't match! So we can't think about being lazy >_<

tial eccentricity at ~ 0.01Hz. We also investigate the bias between the injected and recovered chirp mass when neglecting eccentricity, which increases from $\lesssim 10^{-6}M_\odot$ at $e_i = 0$ to $\gtrsim 10^{-3}M_\odot$ at $e_i = 0.1$. Such systematic bias could be even larger in the full parameter space. It is therefore necessary for searches to take eccentricity into account.

TABLE I: Template bank sizes for GW150914- and GW190521-like events with different parameter spaces.

	Parameter space	GW150914-like	GW190521-like
TianQin	$e_i \in [0, 0.1]$	117202	49943
	$\mathcal{M} \in \mathcal{M}_0 \pm 10\sigma_\mathcal{M}$	3034	4250
LISA	$e_i \in [0, 0.1]$	100403	44867
	$\mathcal{M} \in \mathcal{M}_0 \pm 10\sigma_\mathcal{M}$	2070	3088

Now let's generate template banks with only eccentricity and chirp mass separately, see Table 1 ↑. We need 100,000 templates for eccentricity only?! Also the code was running on the server for a looooooooooong time. Come on, we only consider eccentricity up to 0.1 here!

$e_i = 5\times10^{-4}$ ($e_i = 0.1$) as the smallest distinguishable eccentricity (the upper limit by the current computational cost), which corresponds to the red solid (dashed) line

Leaving aside this for a moment, this eccentricity distribution is interesting, it matches the estimate from previous study, see Fig 3 ↓: the larger the eccentricity, the larger the number of templates needed in a same range

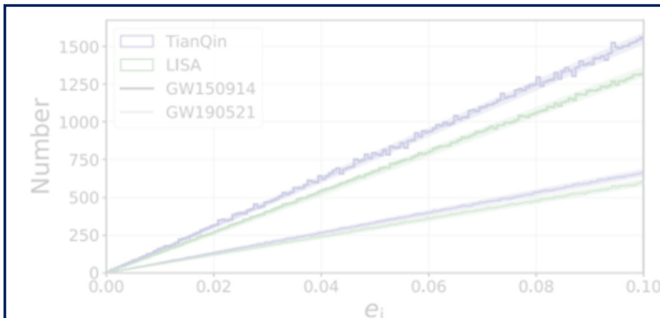


FIG. 3: The distribution of the eccentricity in the archival search template bank. The shaded regions represent the 1σ Poisson fluctuation.

In that case, we can estimate the size of the template bank when considering both parameters: Hundreds of millions of templates. Templates for space detector are already complex and slow to calculate, that'll take ages to calculate!

eccentricity range increases, the full 2D archival search banks are expected to have $N_T \sim \mathcal{O}(10^8)$ templates, if we consider the maximal valid range for `EccentricFD`, i.e. $e_i \in [0, 0.4]$, N_T will be up to $\mathcal{O}(10^9)$.

To evaluate if we have overestimated the magnitude of 2D bank size due to any degeneracy between the eccentricity and the chirp mass [60–62], we generate a 2D

We also did try to calculate a smaller 2D template bank, and the results reinforced the order of magnitude estimate presented above

tiplication of bank sizes that are calculated separately in their parameter spaces. Such results do not change our magnitude estimation of the full 2D archival search bank size. This indicates the challenge of computational cost: an example 2D bank with $e_i \in [0, 0.001]$ includes 13372 templates, and would need $\sim$ 80hr for one core (and 18 GB of memory to cache waveforms) to generate. By slicing the full parameter space along eccentricity and generating the 2D bank in parallel, a bank with $N_T \sim \mathcal{O}(10^8)$ needs $\sim 8\times10^5$ core hours (and $\sim 10^5$GB of memory).

Wait! Are these numbers reliable? What if things aren't that bad and we're overestimating?

See Fig 4 ↓: Let's do a validity and redundancy test

- Validity: pass √
- Redundancy: low, pass √

than 0.97, only 0.44% of them fall lower than 0.97.

So, our conclusions still hold true.

ated bank. We calculate the match between every template in the template bank. An ideal bank will have no redundancy, meaning the matches between all pairs of templates should be smaller than the minimal match threshold. In Fig 4, following the validity test, for each template we present the histogram of the fitting factor, which is calculated on a bank that excludes the template itself. We find that only 6.22% of all templates are redundant. This brings marginal extra computational cost.

IV. From what we have done to painting a rosy picture for the future :)

Numerous studies pointed out that the eccentricity of sBBHs will play a significant role in unveiling their origin.

We've really got archival search working, added eccentricity on top of that, and estimated how much the additional burden is.

We generate one-dimensional template banks for either initial eccentricity or for chirp mass. The upper limit of

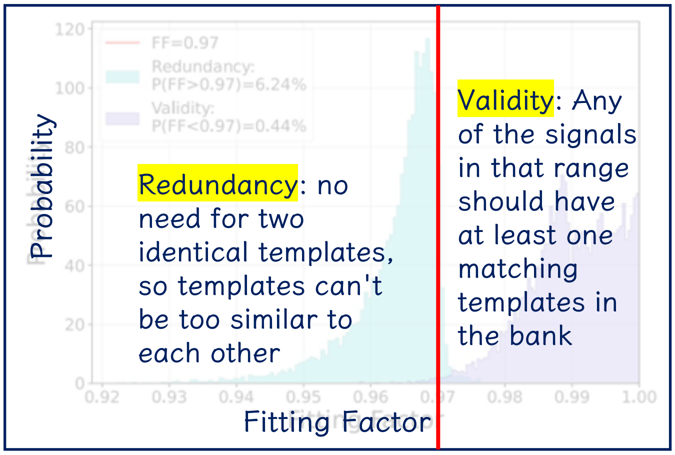


FIG. 4: Validity and redundancy test of the example 2D template bank. The histogram in purple (cyan) shows the result of the validity test (redundancy test). The vertical red line corresponds to the match criteria $M = 0.97$.

People seem to think that with so many parameters of sBBHs, one more (eccentricity) won't be too difficult. But we show that it is not true!

initial eccentricity at a system five years before merger is 0.1. The range of chirp mass is determined by the estimation with the ground-based network. By extrapolating the one-dimensional bank results, we conclude that a two-dimensional eccentric bank will comprise [illegible] $\mathcal{O}(10^{8-9})$ templates, which is $\sim \mathcal{O}(10^5)$ larger compared to the zero eccentricity case, and will require $\sim \mathcal{O}(10^6)$ core hours [and $\sim \mathcal{O}(10^5)$GB of memory] for the pipeline to generate it [also $\sim \mathcal{O}(10^5)$ larger compared to noneccentric case]. This conclusion is verified by a small 2D bank, where [illegible] storage and filtering [illegible] template banks of 10^{8-9} waveforms will therefore be a challenging task, but it is not outside the scope of the expected computational facilities in the late 2030s, and could be further improved with additional optimization of the relevant software techniques.

It's true that we solved a problem, but brought new challenges. Don't worry too much, on the one hand we do need more efficient algorithms, on the other hand computational cost will unlikely to be an issue in the 2030s

Our work demonstrates that this approach can indeed help to distinguish how these sBBH systems are formed

Our work provides a practical solution to the realistic multiband GW observation scenario with which one can determine the formation mechanism of sBBHs with successful archival searches.

It should be noted that we use a nonspinning eccentric waveform model in the paper. It is already known that spin effects are largely negligible during the inspiral[63] phase. [illegible] at all because [illegible] would already be constrained by ground-based facilities. Our space-based archival search would then just search a range of chirp mass and eccentricity values, using the measured black hole spins. It is important to note though that for both ground- and space-based detectors, more precise waveform models will be needed in the future to avoid potential systematic errors [58, 64–67].

But one thing is, we used a waveform with eccentricity but no spin, which is fine in this paper, but more accurate waveforms needed in the future!

One caveat in the study is the duty cycle. We consider ET + dual CE for the ground-based detectors, whereas in reality the duty cycle cannot reach 100%; so the sky localization from realistic future networks might be worse than our calculation. Space-based observatories will also be limited by duty cycles[18, 68]. We leave the detailed calculation to future studies.

Folks, we still have a lot to do :)

V. ACKNOWLEDGMENTS

V. Thank you all!

We are grateful to Xiang-Yu Lyu, En-Kun Li, Jiandong Zhang and Shuai Liu for the helpful discussions. This work has been supported by the Guangdong Major Project of Basic and Applied Basic Research (Grant No. 2019B030302001), the Natural Science Foundation of China (Grant No. 12173104), and the National Key Research and Development Program of China (No. 2020YFC2201400). I.H. acknowledges support from the UK Space Agency through Grant No. ST/X002225/1.

The Shoulders of Giants Here ↓

(With 3 pages more but let's end it here)

[1] [illegible] P. Aufdenberg, Ronald A. Remillard, Mark J. Reid, Ramesh Narayan, and Lijun Gou. The Mass of the Black Hole in Cygnus X-1. *Astrophys. J.*, 742(2):84, December 2011. doi: 10.1088/0004-637X/742/2/84.

[2] Jesus M. Corral-Santana, [illegible] Darias, Franz E. Bauer, Ignacio G. Martinez-Pais, and David M. Russell. BlackCAT: A catalogue of stellar-mass black [illegible] A61, 2016. doi: 10.1051/0004-6361/201527130.

[3] Feryal Özel, [illegible] frey E. McClintock. The Black Hole Mass Distribution in the Galaxy. *Astrophys. J.*, 725(2):1918–1927, December 2010. doi: 10.1088/0004-637X/725/2/1918.

[4] B. P. Abbott et al. Observation of Gravitational Waves from a Binary Black Hole Merger. *Phys. Rev. Lett.*, 116(6):061102, 2016. doi: 10.1103/PhysRevLett.116.061102.

[5] B. P. Abbott et al. Astrophysical Implications of the Binary Black Hole Merger GW150914. [illegible]

818(2):L22, 2016. doi: 10.3847/2041-8205/818/2/L22.

[6] R. Abbott et al. GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run. *arXiv e-prints*, page arXiv:2111.03606, 11 2021. URL https://arxiv.org/ [illegible]

[7] Alexander H. Nitz, Sumit Kumar, Yi-Fan Wang, Shilpa Kastha, Shichao Wu, Marlin Schäfer, Rahul Dhurkunde, [illegible] 4-OGC: Catalog of gravitational waves from compact-binary mergers. *arXiv e-prints*, art. [illegible] 2021. doi: 10.48550/arXiv.2112. [illegible] 06878.

[8] Leor Barack et al. Black holes, gravitational waves and fundamental physics: a roadmap. *Class. Quant. Grav.*, 36(14):143001, 2019. doi: 10.1088/1361-6382/ab0587.

[9] [illegible] Search for Eccentric Binary Black Hole Mergers with Advanced LIGO and Advanced Virgo during their First and Second Observing Runs. *Astrophys. J.*, 883(2):149, 2019. doi: 10.3847/1538-4357/ [illegible]

Thanks for Watching!

Outreach videos about gravitational waves [中文]:

- Gravitational Waves: Starting with 3+1 Questions
- Multimessenger Astronomy: Sharp Ears and Keen Eyes

View the original in: https://humphreywang.github.io/

Font: LXGW WenKai GB / 霞鹜文楷 GB